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POST YIELDING DEFLECTION OF R.C. BEAMS

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ABSTRACT

A method of computing moment curvature relationship for reinforced concrete beams considering contribution of concrete in tension is developed. Normalized Stress-Strain relationship beyond a strain of 0.002 is suggested based upon experimental evidences, for concrete in compression. A computer routine is developed to compute moment-curvature and moment-deflection relationships both for singly and doubly reinforced rectangular sections. Ten beams were tested beyond the yielding of tension steel up to failure. The computed strengths of R. C. Beams at yielding and ultimate stages agree closely with the experimental ones. The computed and experimental moment-deflection curves also compare well. It is concluded that complete moment-curvature characteristics and thus complete behavior of simply supported R. C. Beams can be predicted through all the stages up to failure.

INTRODUCTION

Earthquake is one of the relentless and severe actions, to which a reinforced concrete structure may be exposed. The survival of structure in an earthquake, apart from many other factors, depends upon the energy absorption and moment redistribution capacity of various members. The moment-curvature relationship reveals tremendous information about the energy absorption and moment redistribution capacity of a member. Reinforced concrete being a nonlinear material, poses problems in reasonable prediction of moment-curvature and load-deflection relationships. The creep of concrete further complicates the nature of the problem. If exact member moment-curvature relationships of a structure are known, it is quite easy to predict its performance during an earthquake of known intensity. Some reasonable approximations in the absence of exact load-deformation informations may even work satisfactorily. In the present study moment-curvature relatioship is computed upto collapse of a beam.

STRESS-STRAIN RELATIONSHIP

The normalized stress-strain relationship shown in figure 1 is adopted for concrete in compression. Let

$$y = f(x) \qquad (1)$$

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$$y = \frac{f_c}{f_c'} \tag{2}$$

$$x = \frac{\epsilon_c}{\epsilon_c'} \tag{3}$$

The function $f(x)$ in equation 1 may take one of the two forms expressed by equations 4 and 5 depending upon the value of x, i.e;

$$f(x) = 2x - x^2 \qquad for \quad 0 \le x \le 1 \tag{4}$$
$$f(x) = D_o + D_1 x + D_2 x^2 + D_3 x^3 + D_4 x^4 \qquad for \quad 1 \le x \le 5 \tag{5}$$

Equation 4 is the famous Ritter's Parabola, whereas equation 5 is obtained from a regression analysis of the results from more than 300 compression tests on normal strength concrete with f_c' varying between 2250 Psi to 6575 Psi. These tests were performed under a controlled condition so that cylinders may be loaded to get the descending branch of stress-strain curves. In normal compression test of concrete cylinders using a compression machine, the cylinder is crushed rapidly, because the energy provided by the machine is much much greater than can be absorbed by the concrete specimen. The concrete cylinder may be strained beyond a strain of .002, if the difference between energies supplied and absorbed can be absorbed by some accompanying device. Such a device, a testing frame, was devised at University of Tokyo by You [1]. In this method of testing the unbalanced energy is absorbed by the frame and concrete cylinder is not crushed rapidly. The values of 5 constants in equation 2 found from regression analysis are as follows:

D_o=1.141888, D_1=.114323, D_2=-.359027, D_3=.113423 and D_4=-.0106166

These values are quite different than those found by Hsu [2] from the test made by You[1]. A value of .002 is adopted for ϵ_c'.

The stress-strain relatioship for concrete in tension, shown in Figure 2. is given by the following equations:

$$y = g(x) \tag{6}$$

$$y = \frac{\epsilon_t}{\epsilon_r} \tag{7}$$

$$x = \frac{f_t}{f_r} \tag{8}$$

$$g(x)=x^{m} \quad where \quad m=1 \quad if \quad x \leq 1 \quad and \quad m=-.9 \quad if \quad 1 < x \leq 5 \tag{9}$$

From strain distribution as shown in Figure 3 it may be proved that:

In compression zone
$$x = \frac{\epsilon_{co}\eta}{\epsilon_c c} \tag{10}$$

In tension zone
$$x = \frac{\epsilon_{co}\eta}{\epsilon_r c} \tag{11}$$

ϵ_{co} is extreme fiber compressive strain and ϵ_r is strain coressponding to f_r, peak tensile stress or modulus of rupture of concrete. The values equal to 0.00013 and -0.9 are adopted for ϵ_r, and m respectively as determined by Tahir [3]. The value of y is taken as zero if x > 5 in equation 9.

DERIVATION FOR COMPUTER ROUTINE

A typical section of rectangular R. C. Beam is shown in Figure 3. The condition of equilibrium, in the absence of any axial force, is that compressive axial force equals to tensile axial force. This may be expressed as in the following

$$b f_c' \int_0^c f\left(\frac{\epsilon_{co}\eta}{\epsilon_c c}\right) d\eta + A_s' f_s' = b f_r \int_0^{h-c} g\left(\frac{\epsilon_{co}\eta}{\epsilon_t c}\right) d\eta + A_s f_s \tag{12}$$

And the moment of resistence may be computed as sum of moments due to all the forces about nuetral axis, i.e;

$$M = b f_c' \int_0^c f\left(\frac{\epsilon_{co}\eta}{\epsilon_c' c}\right)\eta d\eta + A_s' f_s'(c-d') + b f_r \int_0^{h-c} g\left(\frac{\epsilon_{co}\eta}{\epsilon_t c}\right)\eta d\eta + A_s f_s (d-c) \tag{13}$$

COMPUTATION OF MOMENT AND CURVATURE

The computer program calculates the moment curvature relationship according to the following step by step procedure.

1: Select a strain in tension steel.
2: Assume depth of nuertal axis, c.
3: Compute $F_c and$ F_t as in equation 12.
4: Check for equilibrium, if satisfied perform step 6.
5: Other wise vary value of c, repeat steps 3 and 4.
6: Compute M from equation 13.

7: Compute curvature $\kappa = \frac{\epsilon_s}{d-c}$.

8: Increase steel strain and repeat steps 2 to 7.

9: Terminate the program if equilibrium is not reached after a certain number of iterations in steps 3 to 5. It indicates collapse of the beam. Compute the deflection according to given span and loading conditions.

It is observed that convergence to equilibrium is achieved quickly in the ascending branch of stress-strain curve. However, it is problematic for descending branch of the curve. Some tolerance value may be specified as difference between compressive and tensile forces. When the difference exceeds the tolerance after sufficient cycles of iteration, it is assumed the beam has reached collapse.

TEST PROGRAM

Ten beams were tested in all. All the members have same cross section and span. First three beams were loaded to failure in four point bending, next three in three point bending loaded at center. Remaining four bemas were tested in three point bending loaded away from center. The section properties and test results are summarized in the Table 1.

Table 1. Properties of Beams

Specimen	b In	h In	d In	d' In	A_s In^2	A'_s In^2	f'_c Psi	f_y Psi
UR-4P-1	6	12	10.8		1.359	-	3250	42450
OR-4P-2	6	12	10.8		2.965	-	3250	42450
DR-4P-3	6	12	10.8	1.1	2.965	1.184	3250	42450
UR-CP-1	6	12	10.8		1.276	-	2950	46700
OR-CP-2	6	12	10.8		2.616	-	2950	46700
DR-CP-3	6	12	10.8	1.1	2.616	1.914	2950	46700
UR-3P-1	6	12	10.8		0.990	-	3675	52650
OR-3P-2	6	12	10.8		2.420	-	3675	52650
DR-3P-3	6	12	10.8	1.1	2.420	1.650	3675	52650
BR-3P-4	6	12	10.8		1.500	-	3675	52650

Table 2. Comparison of Test Results.

Specimen	Ultimate Moment K-in			Ultimte Curvature 10^{-3} /in		Curvature at first Yield 10^{-3} /in		Mode of Failure
	ACI	Computed	Tested	Computed	Tested	Computed	Tested	
UR-4P-1	470.4	485.2	476.0	2.355	2.140	.279	.290	T
OR-4P-2	730.8	767.3	732.0	0.543	0.470	-	-	C
DR-4P-3	1010.6	1035.2	1041.0	1.899	1.671	.316	.320	T
UR-CP-1	473.0	488.9	470.0	2.071	2.090	.316	.330	T
OR-CP-2	660.9	694.4	670.0	0.542	0.530	-	-	C
DR-CP-3	992.4	1065.0	1020.0	2.142	2.180	.300	.280	T
UR-3P-1	441.4	453.9	464.0	2.503	2.320	.300	.290	T
OR-3P-2	788.5	830.5	765.0	0.607	0.650	-	-	C
DR-3P-3	1032.5	1109.7	1087.0	2.133	2.070	.324	.310	T
BR-3P-4	617.9	638.7	620.0	1.950	1.980	.352	.350	T

CONCLUSION

The comparison of Table 2 indicates that a reasonable agreement exists between test and computed results. The procedure presented in this paper is based upon direct sectional analysis of concrete beams. Presently the procedure is for rectangular sections, though it may be extended for any shape of sections with little difficulty. A careful study is needed in continuous beams and frames to apply this analysis to practical problems in framed structures.

ACKNOWLEDGEMENTS

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APPENDIX

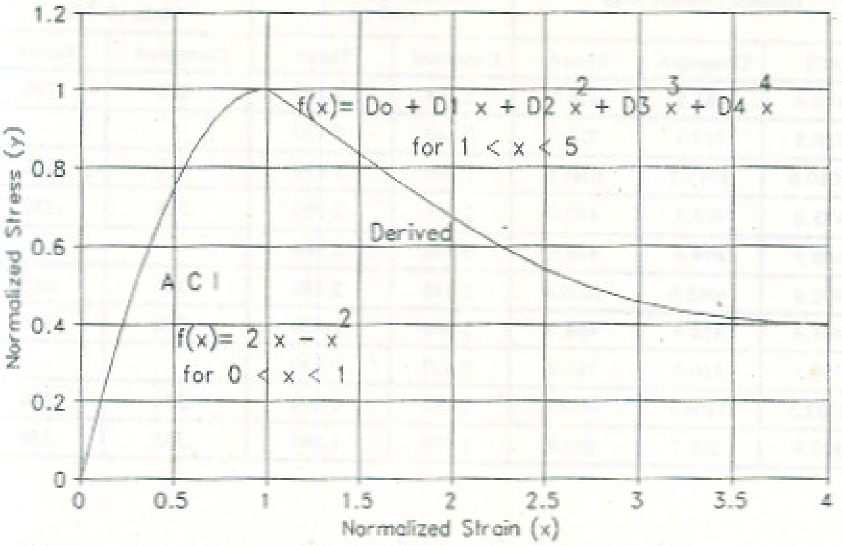

Figure 1 Normalized Stress−Strain Curve

Concrete in Compression

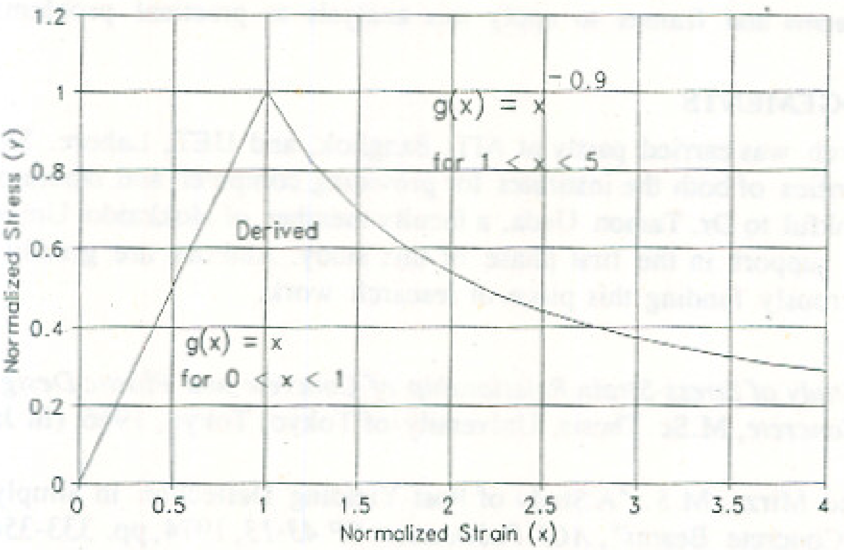

Figure 2. Normalized Stress−Strain Curve

Concrete in Tension

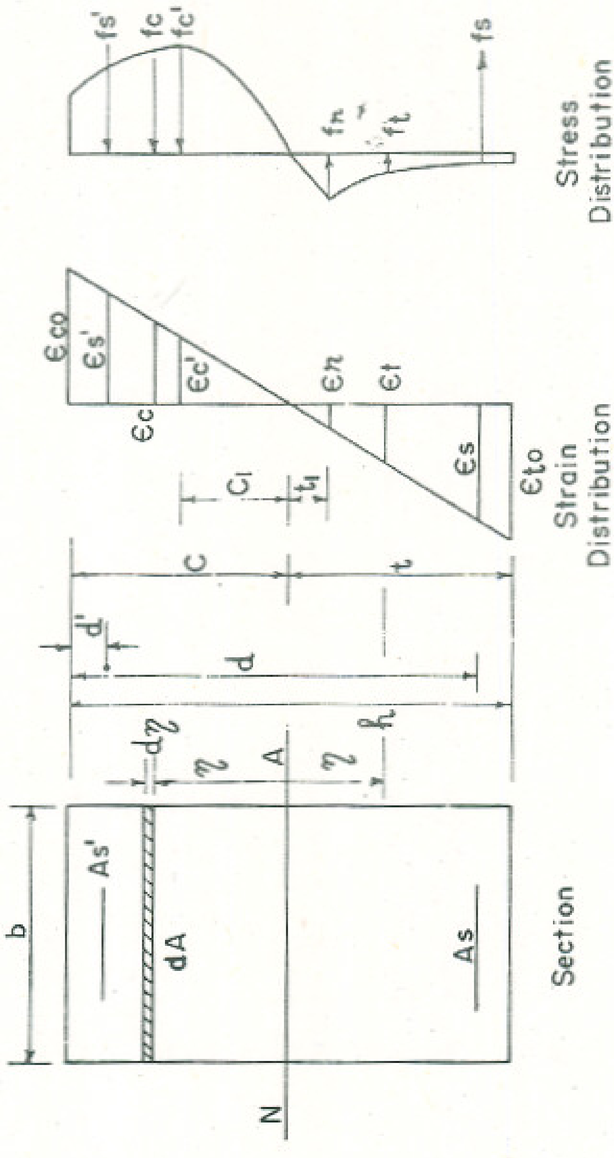

Figure 3-STRESS STRAIN DISTRIBUTION AT SECTION