Paper.438

WIND EFFECT STUDY

ON

AN OGIVAL MASJID DOME

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SYMBOLS USED

A	..	Equation Constant= $1/3\ (Cos^2\phi - Cos^3\phi_0) - (Cos\phi - Cos\phi_0)$
B	..	Equation Constant= $1/2(\phi-\phi_0) - 1/4(Sin2\phi - Sin2\phi_0)$
C	..	Equation Constant= $1/3(Sin^3\phi - Sin^3\phi_0)$
D	..	Equation Constant= $1/2(Sin^2\phi - Sin^2\phi_0)$
F_e	..	Horizontal Component of the External Forces
F_i	..	Horizontal Component of the Internal Forces
$N\phi$	..	Meridional Shell Force ($^k/FT$)
$N\theta$	..	Hoop Shell For-e ($^k/FT$)
$N\phi\theta$	..	Horizontal Plane Shear Force in the Shell($^k/FT$)
Me	..	Moment of External Forces
Mi	..	Moment of Internal Forces
K_1	..	Equation Constant for $N\phi$
K_2	..	Equation Constant for $N\theta$, K_3=Equation Constant for $N\phi\theta$
a	..	Dome depth below right angle
b	..	Distance between the meridian-center and the axis of revolution
d	..	Diameter at the Dome base
$e\phi$	..	Lever arm for F_e
h	..	Over all height of the Dome
r	..	Meridian radius
r_1	..	Perpendicular distance between a point on the meridian and the axis of revolution.
γo	..	Radius, at any parallel circle ϕ , in the horizontal plane
p_H	..	Horizontal wind pressure,p_r =Radial wind pressure
ϕ	..	Meridional Angle, θ :Angle in the parallel circle
ϕ_o	..	Meridional Angle between meridian center and axis of Shell
ϕm	..	Half central Angle = 120 deg.

WIND EFFECT STUDY ON AN OGIVAL MASJID DOME

By
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*Mohammed Azim****

1.0 ABSTRACT

An OGIVAL (pointed) Masjid Dome has been analysed for high velocity Winds. A parameteric study of STRESS-RESULTANTS, under winde effects, has been presented in this paper.

2.0 INTRODUCTION

Quite often it is necessary to choose a dome shape which will make the related structure prominent among the surrounding buildings.

A Masjid, being a place of Workship, is generally always located in the center of a complex or a residential colony .

An Ogival dome is probably the best selection. It is, therefore, that a modest sized reinforced concrete Ogival Dome, having the semi-central angle greater than 90 degrees, has been chosen.

The equations developed for the stress-resultants are parameteric and perfectly general. They have, thus, the advantage that, by varying the parameters, the stress-resultants can readily be obtained for the desired volume.

It is hoped, that, this pattern of work on an Ogival Dome, being very rare in shell literature, would be of interest and aid to the designing engineer.

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3.0 DEVELOPMENT OF THE DESIGN THEORY

3.1 Formation of the Dome-Shape

The shell is generated by rotating about a central axis an arbitrary length of a circular segment having its own center on the largest dome diameter with a given eccentricity with respect to the axis of revolution.

3.2 The parameter "b"

The distance between a meridian-center and the axis of revolution, measured in the horizontal plane containing the meridian center, is indicated as "b". By varying the parameter "b", a family of curves is obtained giving a relative study of the stress-resultants or volume changes.

3.3 Locus of meridional centers

Each meridian, being an arc of a circile, has its center falling within the dome shell. The intersection of the end radii of a meridian is the center of the meridional arc. The diametrically opposite meridians will have their centers on a straight line and on either side of the axis of revolution. This straight line also intersects the axis of revolution in its middle.

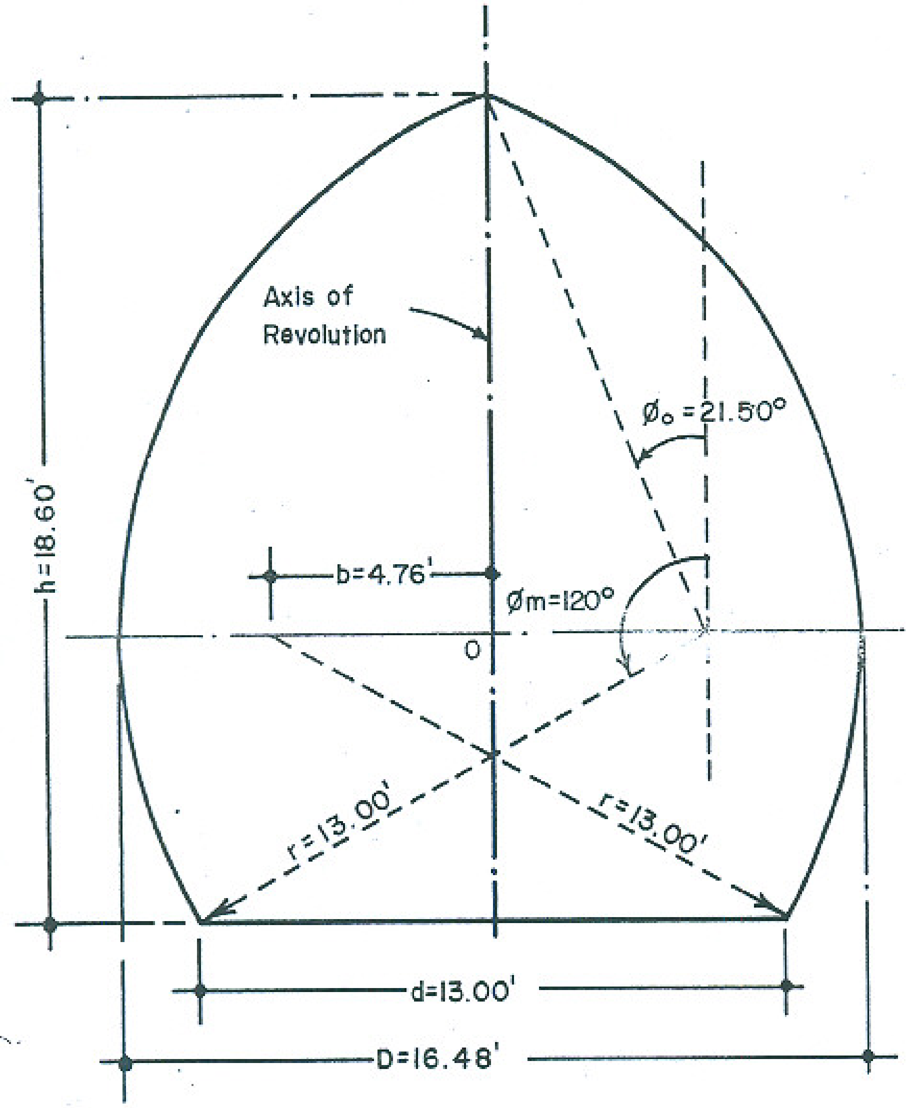
Thus there is formed a circle with the axis of revolution being in its center. This circle is, therefore, a locus of meridional centers. The distance between the axis of rotation and any of these meridional center points is the radius of this circle.

4.0 WIND ANALYSIS OF THE OGIVAL DOME

4.1 Geometry

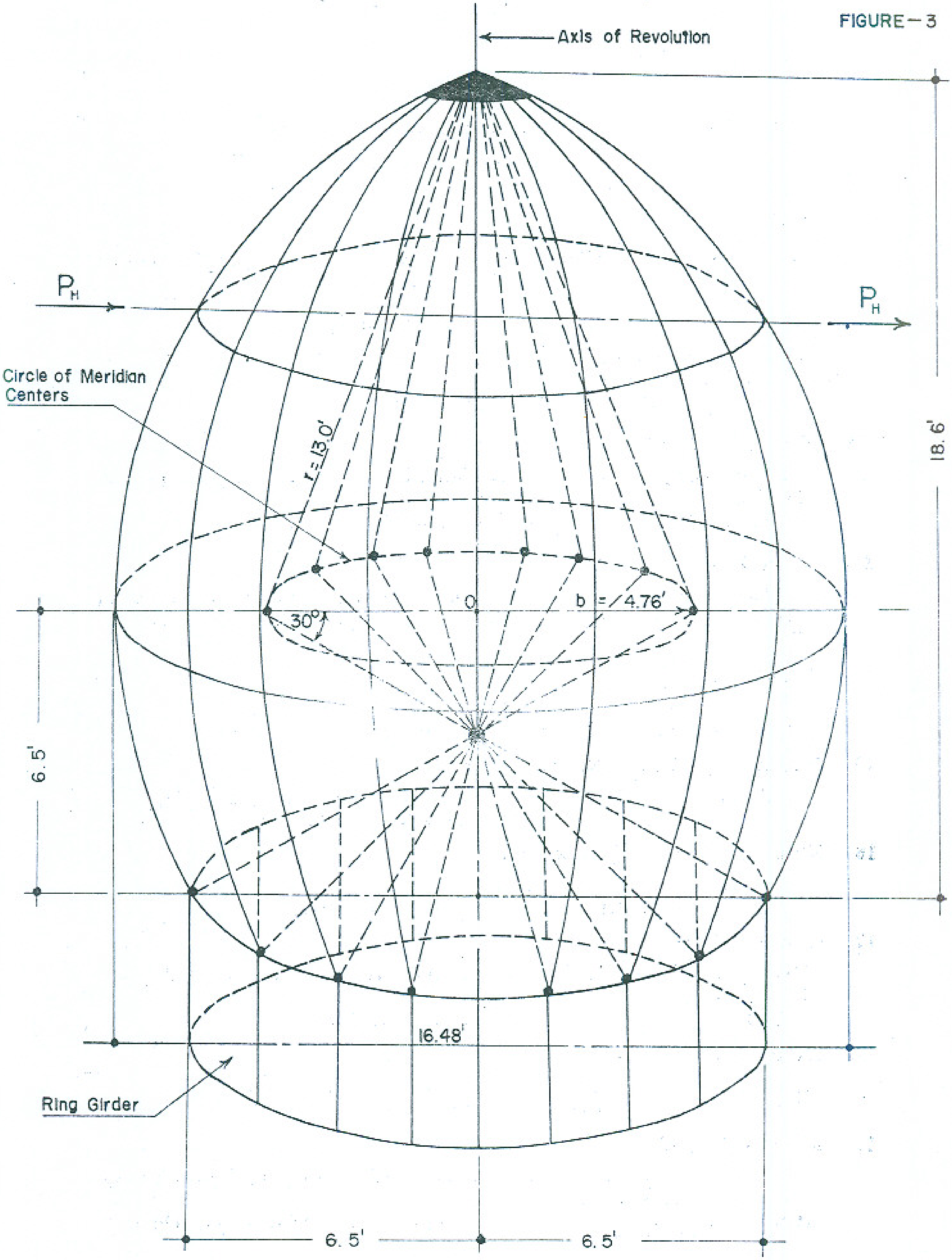
$a = 6.5$ ft., $b = 4.76$ ft.
$D_1 = 16.48$ ft., $d = 13.00$ ft.
$e\phi = r\cos\phi$, $h = 18.6$ ft.

FIGURE – 2



Axis of
Revolution
Ø₀ = 21.50°
h=18.60'
b=4.76'
Øm=120°
O
r=13.00'
r=13.00'
d=13.00'
D=16.48'

FIGURE — 3



OGIVAL DOME

r = 13.00 ft. , $r = r - b/\sin\phi$

r_o $r\sin\phi - b, \phi_o$= 21.50 deg. ϕ_m = 120 deg.

4.2 Assumptions

The wind loading is antisymmetric with respect to the axis of revolution i.e. equal wind pressures exist on the windward side and the leewardside of the dome,.

The direction of the wind pressure p_H is horizontal being in the meridian plane $\theta = 0$

The edge-support conditions are such that only memberane stresses exist in the shell.

5.0 DEVELOPMENT OF THE GENERAL EQUATIONS FOR THE STRESS- RESULTANTS $N\phi$, $N\theta$, $N\phi\theta$

5.1 If a free-body of the dome-shell above any parallel circle at ϕ is considered, three statical equations are available,

$$\Sigma F_x = 0 \quad \Sigma F_r = 0, \quad \Sigma M\phi = 0$$

where,

$\Sigma F_x = \Sigma F_e + \Sigma F_i = 0$, summation of all the horizontal components of the external and internal forces, acting above the Horizontal Plane at "ϕ" , on the free-body,

$\Sigma F_r = \Sigma \frac{N\phi}{r} + \Sigma \frac{N}{r_1} + \Sigma p_r = 0$, summation of radial components of the external and internal forces for the upper free-body at "ϕ"

$\Sigma M\phi = \Sigma M_e + \Sigma M_i = 0$, summation of moments of F_r

and F_x components about the line $\theta = \pm\frac{\pi}{2}$ in the free-body parallel circle at "ϕ"

5.2 Point 0_1 is the center of the meridian i-j-k which has been obtained by the intersection of the end radii $i-0_1$ and $k-0_1$ and it lies in the meridian plane i-j-k-l-m inclined at θ. At the same time, it also lies in the horizontal plane of parallel circle at $\phi = 90^o$.

5.3 The radial component of the wind pressure 'p_r' passing through the center O_1 develops a horizontal component in the horizontal plane $\phi = 90^o$ equal to $p_H \sin^2\phi \cos^2\theta$ a distance $r\cos\phi$ from the plane parallel circle at "ϕ ". p_r also has a vertical component = $p_H \sin\phi\cos\phi \cos\theta$ contained in the meridian plane at θ(plane i-j-k-l-m)

5.4 Horizontal components of internal forces:

* $N\phi$ has a horizontal component:
  $+ N_\phi \cos\phi \cos\theta \; r_o d\theta$

* $N\phi\theta$ has a horizontal component:
  $-N_{\phi\theta} \sin\theta \; r_o \; d\theta$

5.5 Vertical components of internal forces:

* $N\phi$ has a vertical component:
  $N_\phi \sin_\phi r_o d\theta$

5.6 Radial components of internal forces:

* N_ϕ has a radial component
  $- N_\phi r_o \; d\theta d\phi$

* $N\theta$ has a radial component
  $- N\theta \; r \; d\phi d\theta \sin\phi$

5.7 The vertical component of N_ϕ ($N_\phi \sin_\phi r_o \; d\theta$) produces clock-wise moment about the line $\theta = \pm\pi/_2$, with a lever arm = $r_o \cos\theta$

5.8 The horizontal component of the wind pressure, $p_H \sin^2\phi \cos^2\theta \; r_o \; d\theta \; r \; d\phi$, produces counter-clockwise moment about the line $\theta = \pm\pi/_2$ with a lever arm $e_\phi = r\cos\phi$

5.9 The vertical component of the radial wind pressure is equal to $p_H \sin\phi\cos\phi\cos\theta r_o d\theta r d\phi$ & it acts vertically at its relevant meridional center located in the parallel circle plane at $\phi = 90^o$. And it produces a clock-wise moment at the free-body with a lever arm of $b\cos\theta$ on the upper free-body at ϕ.

FIGURE - 4a

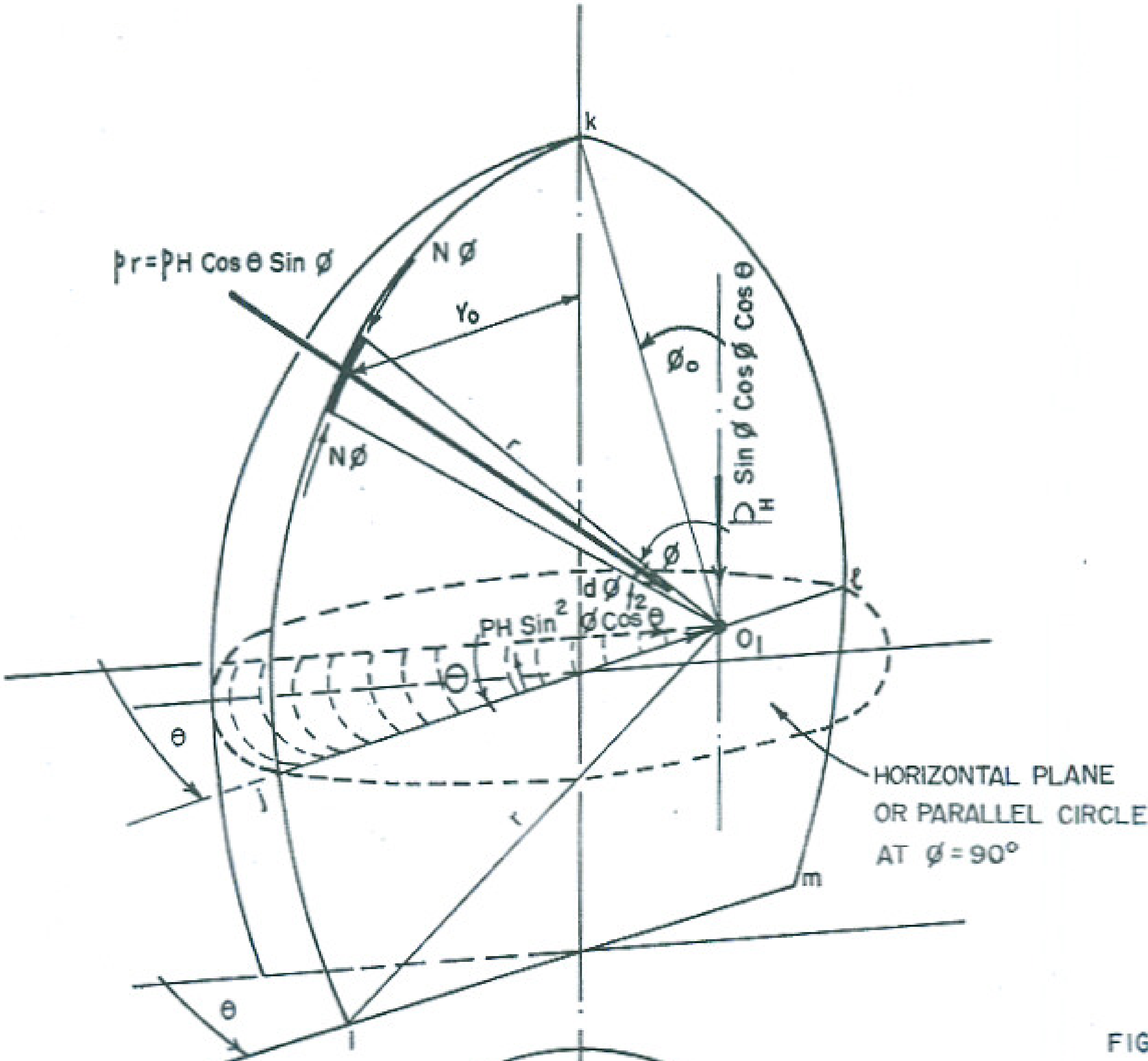
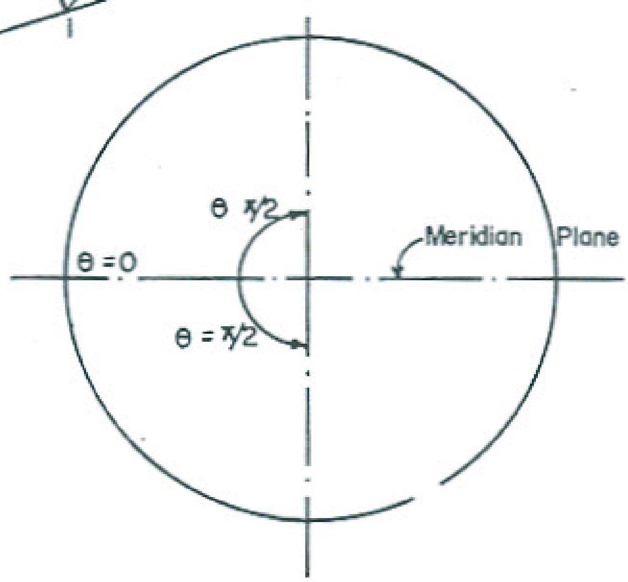


FIGURE - 4b



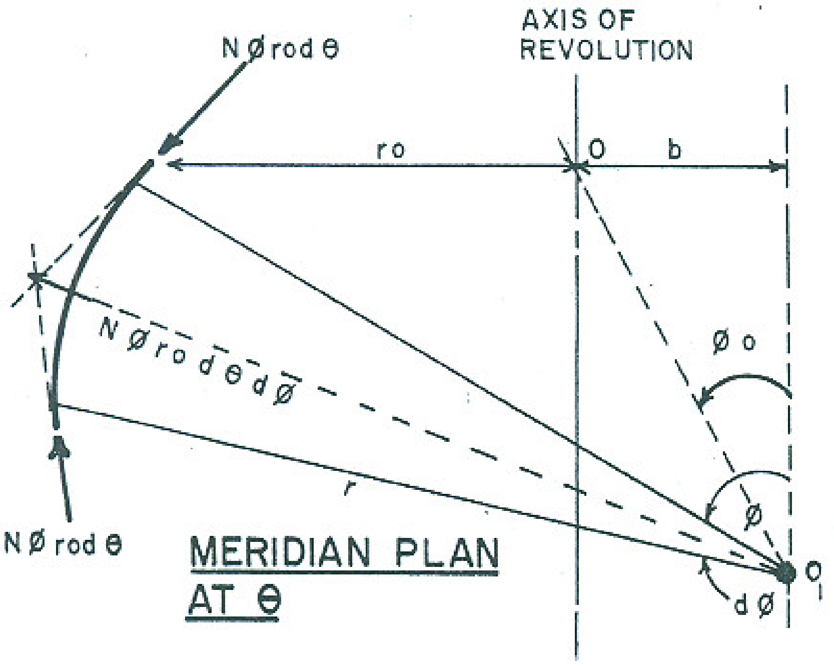


FIGURE 5-a

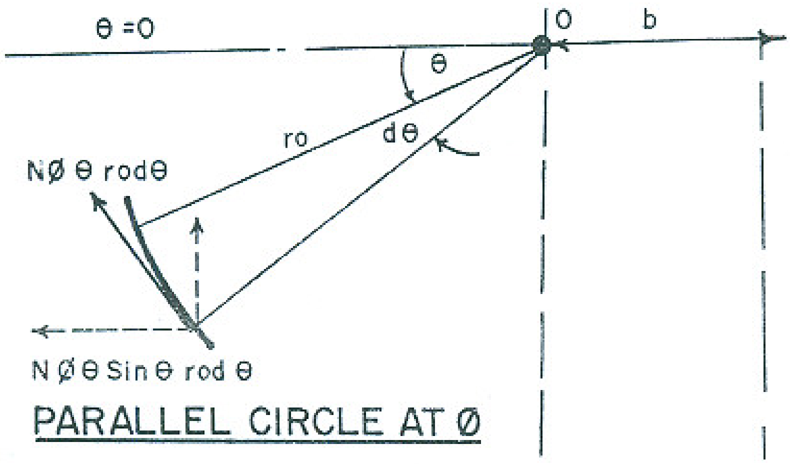


FIGURE 5-b

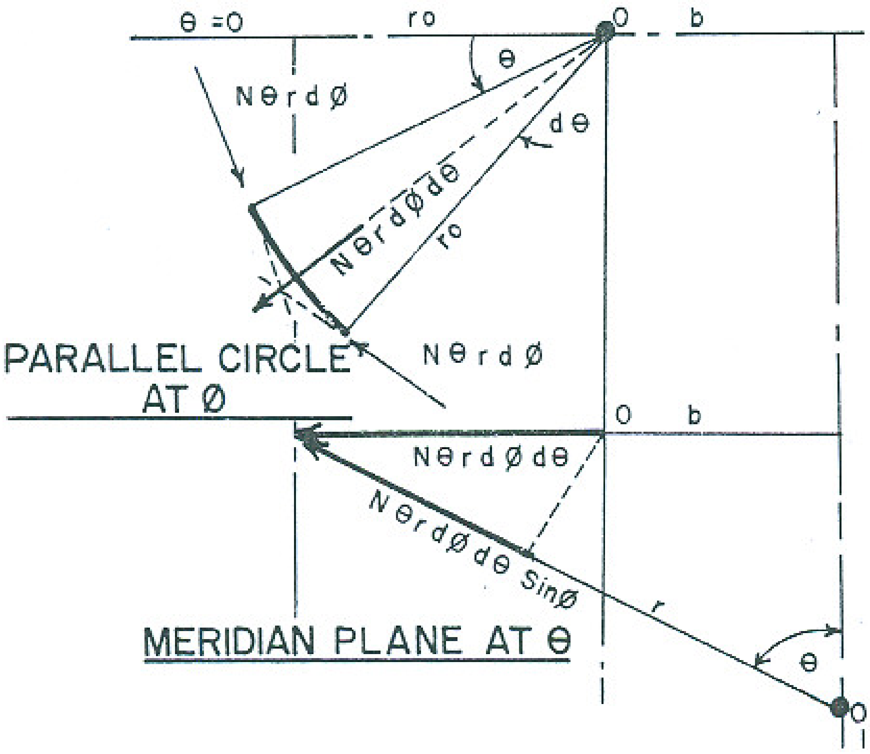


FIGURE 5-C

It is to be noted that the stress-resultants are a direct function of "ϕ" and "θ" hence,

* $N\phi$ varies directly with ϕ and $\cos\theta$

$\therefore N\phi = k_1 \cos\theta$, where k_1 is a function of ϕ alone,

* $N\theta$ varies directly with ϕ and $\cos\theta$

$\therefore N\theta = k_2 \cos\theta$, where k_2 is a function of ϕ alone

* $N\phi\theta$ varies directly with ϕ and $\sin\theta$

$\therefore N\phi\theta = k_3 \sin\theta$, k_3 is a function of ϕ alone

5.10 Evaluation of ΣF_e

$$\Sigma F_e = + 4 \int_{\phi_o}^{\phi} \int_{0}^{\pi/2} p_H \sin^2\phi \cos^2\theta \, r_o d\theta r d\phi$$

After integration and simplification

$$\Sigma F_e = \pi p_H r^2 \{(\cos^3\phi - \cos^3\phi_o)1/3 - (\cos\phi - \cos\phi_o)\}$$

$$-\pi p_H b r \{1/2(\phi - \phi_o) - 1/4(\sin 2\phi - \sin 2\phi_o)\}$$

5.11 Evaluation of ΣF_i

$$\Sigma F_i = + 4\{\int_0^{\pi/2} N_\phi \cos\phi \cos\theta r_o d\theta - \int_0^{\pi/2} N_{\phi\theta} \sin\theta r_o \, d\theta\}$$

substituting for $N_\phi = k_1 \cos\theta$ and $N_{\phi\theta} = k_3 \sin\theta$ and after integration and simplification

$$\Sigma F_i = +\pi r_o (k_1 \cos\phi - k_3) \quad \text{where, } r_o = (r\sin\phi - b)$$

5.12 Evaluation of ΣM_e

If a free body of the upper dome portion, at any meridional angle ϕ, is taken all the wind force acting at this upper portion will pass through their respective meridional centers located in the parallel circle(horizontal) plane at ϕ= 90 deg.

These radial wind effects(= $p_H \sin\phi \cos\theta$) can be resolved into the orthogonal components in the parallel circle plane ϕ= 90 deg., as shown in the Fig. 6, at their respective centers.

Considering the moment equilibrium of the upper free-body, it is seen that the resultant of the horizontal wind components F_e being always in the plane ϕ = 90 deg. produces a counter-clockwisen moment at the free-body with a lever-arm = $r\cos\phi$. While those vertical, produce a clockwise moment as has been stated before.

Thus, $\Sigma M_e + \Sigma M_i = 0$

$$\text{or} - \Sigma F_e(r\cos\phi) + 4\int_0^{\pi/2}\int_{\phi_0}^{\phi} p_H \sin\phi\cos\phi\cos\theta r_o d\theta r d\phi(b\cos\theta) + $$

$$+ 4\int_0^{\pi/2} N_\phi \sin\phi r_o d\theta(r_o\cos\theta) = 0$$

substituting for ΣF_e its value from page438and for $N_\phi = k_1 \cos\theta$

$$-\{\pi p_H r^2 [1/3(\cos^3\phi-\cos^3\phi_o)-(\cos\phi-\cos\phi_o)]-\pi p_H br [1/2(\phi-\phi_o)$$

$$-1/4(\sin2\phi-\sin2\phi_o)]\} r(\cos\phi)+4\int_0^{\pi/2}\int_{\phi_0}^{\phi} p_H \sin\phi\cos\phi\cos\theta r_o d\theta r d\phi(b\cos\theta)$$

$$+ 4\int_0^{\pi/2}(k_1 \cos\theta) \sin\phi r_o\, d\theta(r_o \cos\theta) = 0$$

Integrating and Simplifying

$$- p_H r^2 \{1/3(\cos^3\phi-\cos^3\phi_o)-(\cos\phi-\cos\phi_o)\} - p_H br\{1/2(\phi-\phi_o)$$

$$-1/4(\sin2\phi-\sin2\phi_o)\} (r\cos\phi)+bp_H r^2 \{1/3(\sin^3\phi-\sin^3\phi_o)\}$$

$$-b^2 p_H r\{(\sin^2\phi-\sin^2\phi_o)\} + k_1\sin\phi(r\sin\phi-b)^2 = 0$$

solving for k_1

$$k_1 = +p_H r^3\cos\phi(A)/(r_o^2\sin\phi)-bp_H r^2\cos\phi(B)/(r_o^2\sin\phi)-bp_H r^2(C)/(r_o^2\sin\phi)$$

$$+b^2 p_H r(D)/r_o^2\sin\phi \quad \text{where, } A=1/3(\cos^3\phi-\cos^3\phi_o)-(\cos\phi-\cos\phi_o)$$

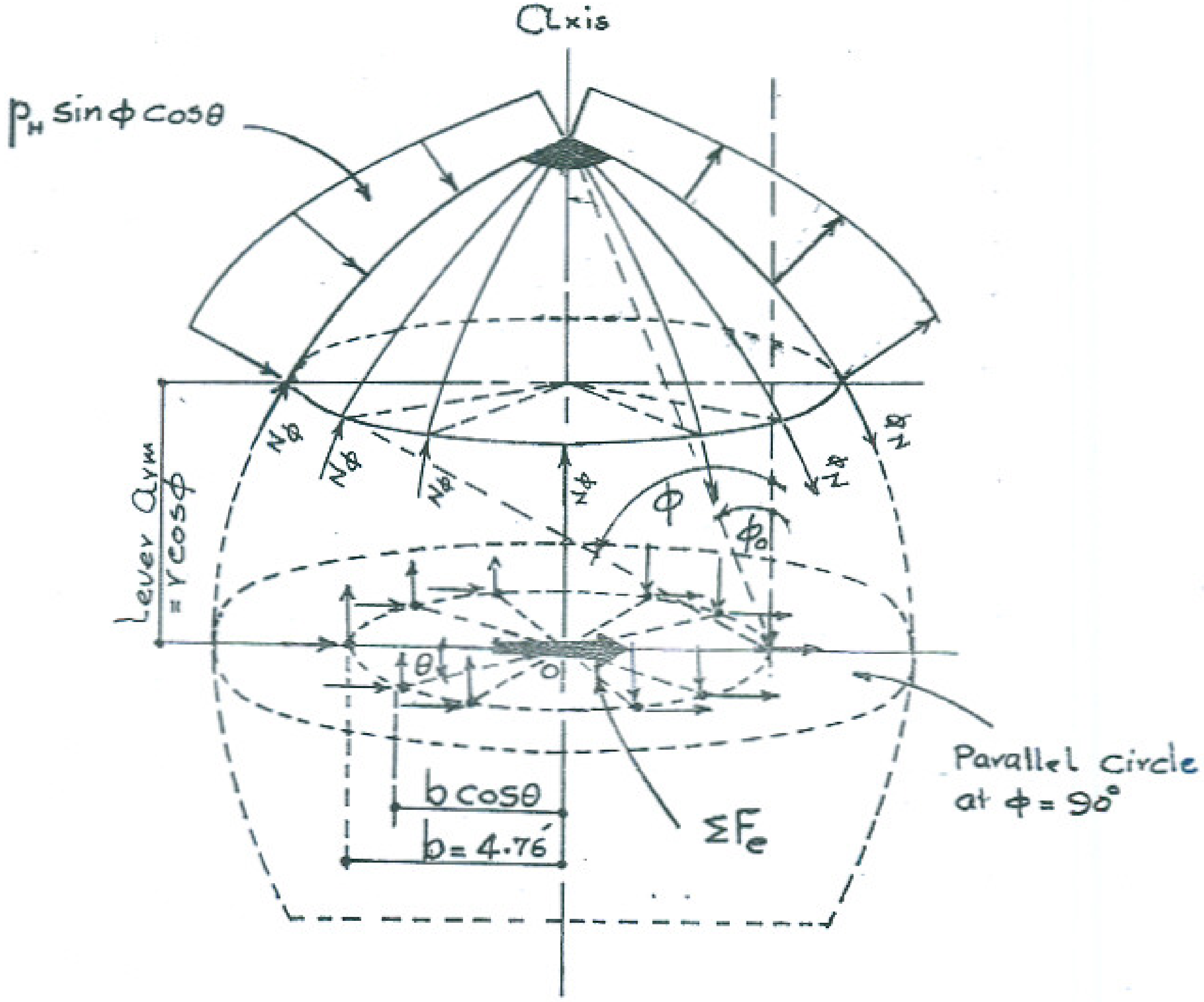
$$B=1/2(\phi-\phi_o)-1/4(\sin2\phi-\sin2\phi_o)$$

$$C=1/3(\sin^3\phi-\sin^3\phi_o) \quad \text{and}$$

$$D=1/2(\sin^2\phi-\sin^2\phi_o)$$

Now by changing the value of "ϕ" in k_1, meridional variation of $N\phi$ is obtained

FIGURE NO 6



Axis
$P_H \sin\phi \cos\theta$
N_ϕ
Lever Arm
$= r\cos\phi$
ϕ
ϕ_0
θ
O
$b\cos\theta$
$b = 4.76'$
ΣF_e
Parallel circle
at $\phi = 90°$

5.13 Evaluation of N

$N\phi/r + N\phi/r_1 - p_r = 0$

substituting for $N\phi = k_1\cos\theta$ and for $p_r = p_H\sin\phi\cos\theta$ and noting that $r_1 = (r\sin\phi - b)$

$N_\theta = (p_H - k_1/r\sin\phi)(r\sin\phi - b)\cos\theta$

5.14 Evaluation of $N_{\phi\theta}$

Summing the horizontal components of internal and external forces at the free-body at ϕ

$\Sigma F_e + \Sigma F_i = 0$, substituting the values for ΣF_e and ΣF_i from page 279.

or $\pi p_H r^2(A) - \pi p_H b\, r(B) + \pi(k_1 r_o\cos\phi - k_3 r_o) = 0$

or $k_3 = k_1\cos\phi + p_H r^2\ (A)/r_o - p_H br(B)\ /r_o$

And $N_{\phi\theta} = k_3\sin\theta = \{k_1\cos\phi + p_H r^2\ (A)/r_o - p_H br(B)/r_o\}\ \sin\theta$

where $r_o = (r\sin\phi - b)$ and the value of k_1 is indicated on page 280.

The three equations developed for the three stress-resultants N_ϕ, N_θ and $N_{\phi\theta}$ are again listed below for ready reference.

$N_\phi = k_1\cos\theta = \{p_H r/(r\sin\phi - b)^2\sin\phi\}\{Ar^2\cos\phi - Bbr\cos\phi - Cbr + Db^2\}\cos\theta$ .I

$N_\theta = k_2\cos\theta = \{(p_H - k_1/r\sin\phi)(r\sin\phi - b)\}\cos\theta$ II

$N_{\phi\theta} = k_3\sin\theta = \{k_1\cos\phi + p_H r^2 A/(r\sin\phi - b) - p_H brB/(r\sin\phi - b)\}\sin\theta$ ...III

The values of these stress-resultants will be computed for the critical wind side.

Two curves will be drawn for each of the above forces: one for the meridional variation and the other for the θ -variation.

The values indicated have been obtained using the above equations.

Then, on the same axes a set of dotted curves have been added, obtained by varying the parameter "b"(=3.5 Ft.).

The trigonometric functions which are a part of the equations developed for the evaluation of stress-resultants, under wind effect, in the present Ogival-Dome case, having the circular-are meridians, are tabulated below for the meridional angle "ϕ" in the range $\phi = 30^o$ to $\phi = 120^o$. These functions have been indicated, in the present paper, under the constants A,B,C, and D. Their usefulness is limited to the present example being computed for a constant value $\phi_o = 21.5$ degree.

$$A = 1/3(\cos^3\phi - \cos^3\phi_o) - (\cos\phi - \cos\phi_o),$$
$$B = 1/2(\phi - \phi_o) - 1/4(\sin 2\phi - \sin 2\phi_o)$$
$$C = 1/3(\sin^3\phi - \sin^3\phi_o),$$
$$D = 1/2(\sin^2\phi - \sin^2\phi_o).$$

ϕ Deg.	A	B	C	D
10	--	--	--	--
20	--	--	--	--
30	.0125	.0282	.02525	.0578
40	.0450	.0857	.07211	.1394
50	.1077	.1730	.13340	.2262
60	.2036	.2899	.20010	.3078
70	.3333	.4330	.26010	.3743
80	.4900	.5955	.30190	.4177
90	.6619	.7682	.31690	.4328
100	.8340	.9410	.30190	.4177
110	.9900	1.1035	.26010	.3743
120	1.1200	1.2466	.20010	.3078

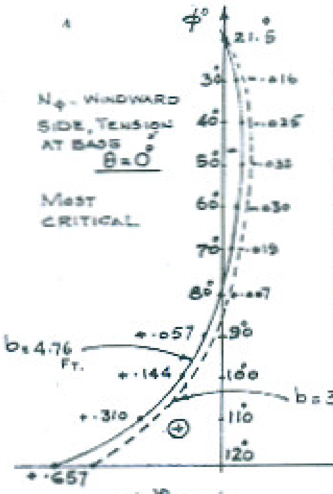
The values indicated on the curves are for b = 4.76 Ft.

REFERENCES

1. Timoshenko, S.P., and Woinowsky-Krieger; "Theory of Plates and Shells". 2d ed., McGraw-Hill Book Company,New York, 1959.

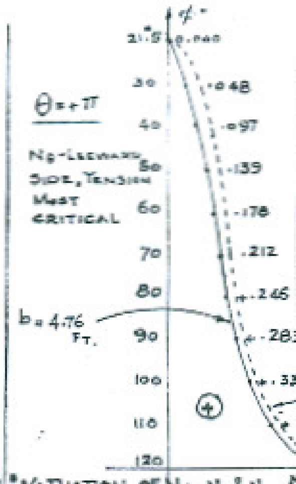
2. Billington, D.P., "Thin Shell Concrete Structure"., McGraw-Hill Book Company,New York, 1965.

3. Harry Kraus, "Thin Elastic Shells". John. Wiley & Sons, Inc. New York , 1967.



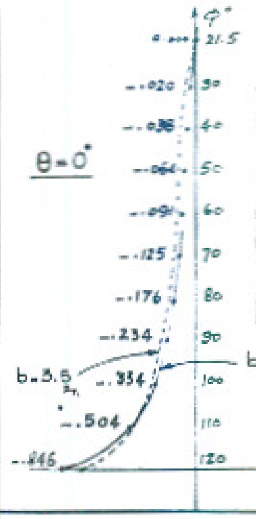
φ°	Nφ K/FT
21.5	0.000
30	-.024
40	-.033
50	-.041
60	-.039
70	-.029
80	-.005
90	+.039
100	+.117
110	+.259
120	+.536

VALUES FOR b=3.5 FT.



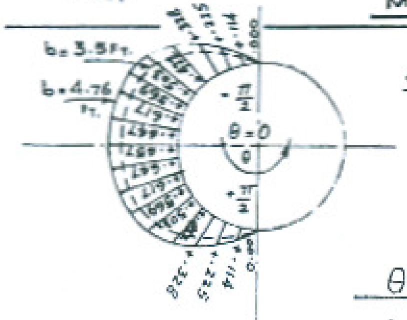
φ°	Nθ K/FT
21.5	0.000
30	+.079
40	+.127
50	+.167
60	+.206
70	+.241
80	+.276
90	+.314
100	+.364
110	+.446
120	+.602

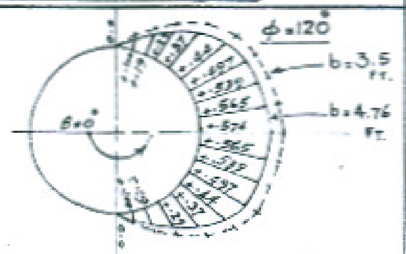
VALUES FOR b=3.5 FT.

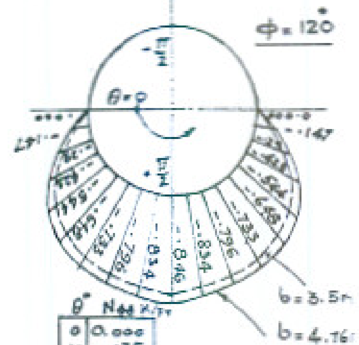


φ°	Nφθ
21.5	0.000
30	-.029
40	-.054
50	-.075
60	-.102
70	-.135
80	-.180
90	-.243
100	-.337
110	-.490
120	-.780

MERIDIONAL VARIATION OF Nφ, Nθ & Nφθ







θ-VARIATION OF Nφ, Nθ & Nφθ AT φ=120°

MOST CRITICAL CASE

VALUES FOR b=3.5 FT. φ=120°

θ°	Nφ K/FT
0	+.536
10	+.528
20	+.506
30	+.464
40	+.411
50	+.345
60	+.268
70	+.186
80	+.093
90	0.000

VALUES FOR b=3.5 FT. φ=120°

θ°	Nθ K/FT
0	+.602
10	+.593
20	+.566
30	+.522
40	+.462
50	+.387
60	+.301
70	+.206
80	+.105
90	+.000

VALUES FOR b=3.5 φ=120°

θ°	Nφθ K/FT
0	0.000
10	-.135
20	-.266
30	-.390
40	-.500
50	-.597
60	-.675
70	-.733
80	-.768
90	-.780